Koch Curve Polar Coordinate Transform for UWB Antenna Applications

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Abstract

Fractal antennas allow for the design of Ultrawide-band (UWB) antennas with a reduced footprint; we present a novel technique for generating antenna geometries which extend the Koch snowflake to angles other than 60 degrees. This is achieved by generating a Koch curve of an arbitrary indentation angle and then circumscribing the curve about the origin. Using our method, we achieve a 36 percent footprint area reduction compared to a traditional Koch snowflake antenna with a center frequency of 4.1 GHz.

1. Introduction

The self-similar characteristics of fractal antennas allow unique multi-band characteristics that are of importance in UWB applications [1]. As fractal iterations increase, the area of the antenna approaches a finite limit while the perimeter of the fractal theoretically diverges to infinity; this yields the *space-filling property* of fractals [2]. The space-filling property allows a fractal antenna to have a large *electrical length* within a compact form factor [3].

The Koch curve is one of the most commonly-used fractal antenna geometries because it demonstrates excellent efficiency at sizes approaching the theoretical limit for small antennas [2].

The frequency characteristics of Koch curve antennas are significantly affected by the fractal dimension of the geometry, which is in turn affected by several factors: the iteration, the indentation angle, and the scale. The fractal iteration is an effective design parameter for the Koch curve antenna; as the iteration increases, the first resonant frequency decreases and the harmonics shift nearer to the resonant frequency. Similarly, the indentation angle can be modulated to achieve desired frequency characteristics [4].

A Koch curve is created by iteratively transforming a line segment into 4 equal line segments, each $\frac{1}{4}$ the size of the initial segment, which are arranged such that an equilateral triangle is added to the center of the line. A conventional Koch snowflake is created by performing the aforementioned iteration process on the sides of an equilateral triangle. Because the initiator shape for a Koch snowflake is an equilateral triangle instead of a simple line as in the Koch curve, the indentation angle can not be modulated in a Koch snowflake as it can be in a Koch curve.

This work provides an analytical framework for the angular transformation of an arbitrary linear Koch curve into a closed snowflake geometry that allows designers more control over frequency and bandwidth without compromising the overall footprint. For our work we use inkjet printing as it decreases fabrication time, and allows direct modification of the design since the antenna can go from simulation to fabrication in a few steps. The Koch snowflake antennas are made of a conductive ink printed directly on a polyimide-based dielectric substrate. Printed antennas have become low cost to manufacture, compact form factor, and high-frequency characteristics [5]. Examples of the use for this type of inkjet-printed antennas on light and flexible substrates are mobile devices that require lightweight antenna elements that will not easily break if bent [6]. Other applications are low-cost IoT devices, and wearable RF devices that can be flexed, bent, and manipulated without damage or plastic deformation [7].

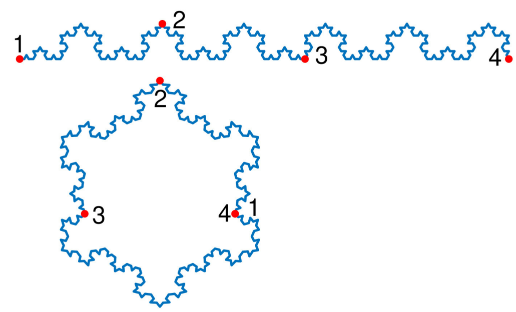

Figure 1: Visualization of transformation procedure of Koch curve to polar Koch snowflake.

2. Polar Transformation of Koch Curve

2.1. Theoretical Modeling

The circumscription process begins with a Koch curve in Cartesian coordinates generated from an initiating line segment of length L. The Koch curve iteration process is performed on this line segment with a given indentation angle until the desired curve is reached.

Let K_n be the set of all points in the n$^{\text{th}}$ iteration Koch curve. We propose a transformation $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ on $K_n \subset$

$\mathbb{R}^2$, defined as follows.

$$\forall p = (a, b) \in K_n$$
$$T(p) = (b + R)\left(\cos\left(\frac{2\pi}{L}a\right), \sin\left(\frac{2\pi}{L}a\right)\right)$$

This can be intuitively understood as the x-axis wrapping around the unit circle in a counter-clockwise direction so that the first and last points of the curve meet, thus forming a closed geometry and can be visualized in Figure 1. The resultant polar functions, $r = r(\hat{y})$ and $\theta = \theta(\hat{x})$, are then mapped to Cartesian coordinates for simplicity of plotting and manufacturing with the transformation $C : \mathbb{R}^2 \mapsto \mathbb{R}^2$ given as;

$$x = r\cos\theta \tag{1}$$
$$y = r\sin\theta \tag{2}$$

The composite transforms $x(\hat{x}, \hat{y})$ and $y(\hat{x}, \hat{y})$, are given below.

$$x = (\hat{y} + R)\cos\left(\frac{2\pi}{L}\hat{x}\right) \tag{3}$$
$$y = (\hat{y} + R)\sin\left(\frac{2\pi}{L}\hat{x}\right) \tag{4}$$

Figure 2 shows that the return loss of the circumscribed 3rd iteration 60° Koch curve is very similar to that of a Koch snowflake with the same height, angle, and iteration. For the base case, a 60° snowflake, the polar transformation method generates a snowflake with almost identical perimeter and area characteristics as a conventional Koch snowflake. This suggests that the polar transformation method is a valid extension of the traditional Koch snowflake.Furthermore, as can be seen in Figure 3, the area and perimeter characteristics are similar for both a standard Koch snowflake and the transformation described in this work.

2.2. Design & Simulation

There are several new parameters made available to the designer by the polar transformation method which were not previously available using a simple Koch snowflake, the most significant being the indentation angle. The indentation angle strongly determines the fractal dimension of the radiating patch geometry, which in turn inversely affects the frequency response [4]. As the indentation angle decreases, the center frequency also decreases and the additional resonant frequencies shift closer to one another. Conversely, decreasing the angle has the opposite effect, moving the resonant frequencies further apart and higher.

Increasing the fractal iteration of a conventional Koch snowflake antenna decreases the resonant frequency, increases the quality factor, and increases the radiation resistance, but with diminishing returns [8]. The same trend is reflected in simulations of the polar Koch geometries. The radius of the polar circumscription affects the scale of the patch geometry which in turn has an inverse relationship with resonant frequency.

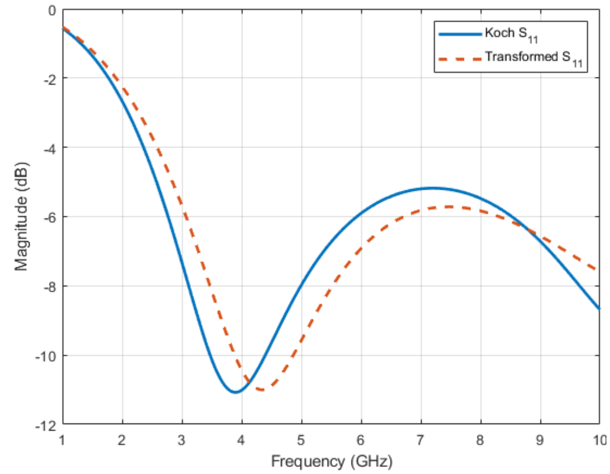

Figure 2: S_{11} characteristics of Koch snowflake vs. wrapped Koch snowflake for 60° indentation.

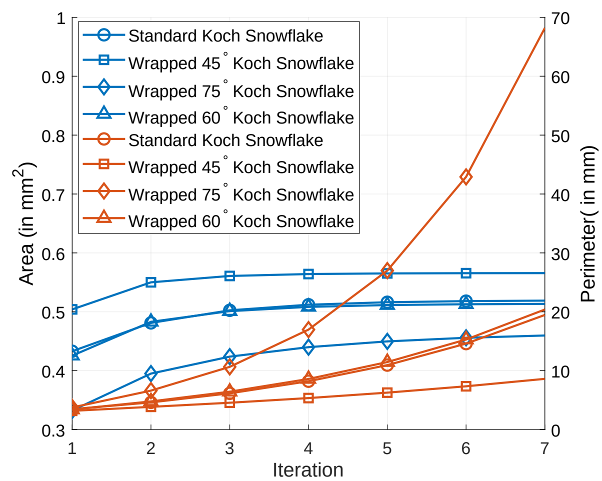

Figure 3: Area plot(blue lines) and Perimeter(orange lines) plots of a various wrapped Koch snowflakes in comparison to a standard Koch snowflake.

We designed antennas using the proposed method which were printed with silver ink on Kapton polyimide film as shown in Figure 7 and measured using an Agilent PNA N5230A Vector Network Analyzer. The ground patch was made small so that the effect of the radiating patch geometries could be isolated from ground plane interactions.Inkjet printing simplifies the antenna fabrication process keeping it low-cost. The desired pattern is written directly onto the substrate followed by a rapid curing thermal treatment. For the inkjet-printing of the antenna area we used a conductive ink based on silver salt particles. After curing at 140 °C for 10 minutes the ink forms a conductive layer of the printed pattern. The printer used for this work is a BotFactory SV2 PCB printer.

The measured results in Figure 5 show that the indenta-

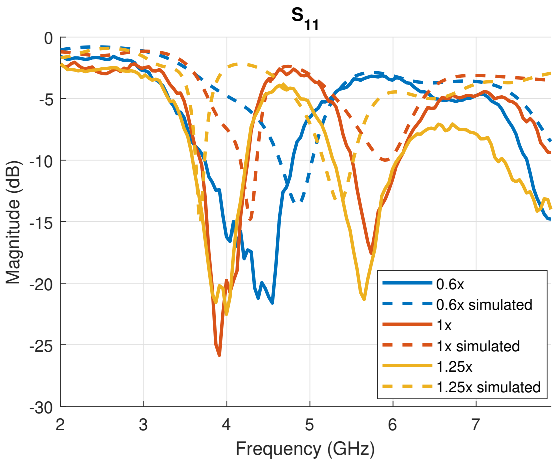

Figure 4: Measured S_{11} parameters of proposed geometry for various scales

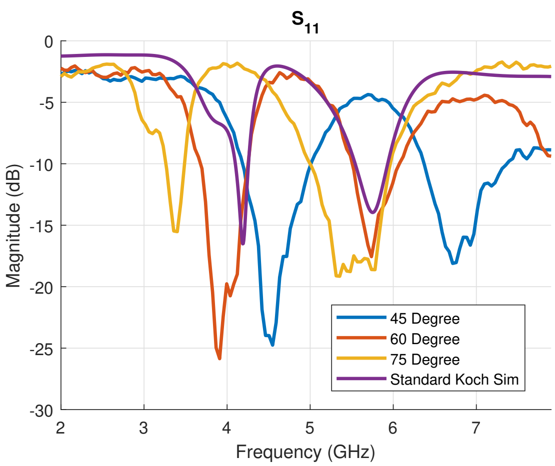

Figure 5: Measured S_{11} parameters of proposed geometry for various indentation angles & simulated S_{11} for traditional Koch snowflake

tion angle effectively sets the multi-band characteristics and resonant frequencies as expected. The measured frequency characteristics for the 60° polar Koch geometry are similar to the simulated characteristics for the standard Koch snowflake. Additionally, the 60° transformed and standard snowflakes have very similar areas and perimeters through various iterations, suggesting that the polar transformation method is a valid extension of the standard Koch snowflake, maintaining the same radiative and geometric features. Figure 4 shows that the measured results for different scales of 60° polar snowflakes have the same pattern as in simulation.

Simulations of standard Koch snowflake antennas and polar Koch antennas with an angle of 75° which are designed to have the same primary resonant frequency demonstrate the area-saving capabilities of the method laid out in this work. The extent of these savings can be seen in Figure 6. The area savings in this instance appear to diminish as the resonant frequency increases because as the radiating patch becomes smaller, the feed lines become dominant. In order to maintain a fair comparison, the feed lines were not modified between resonant frequencies.

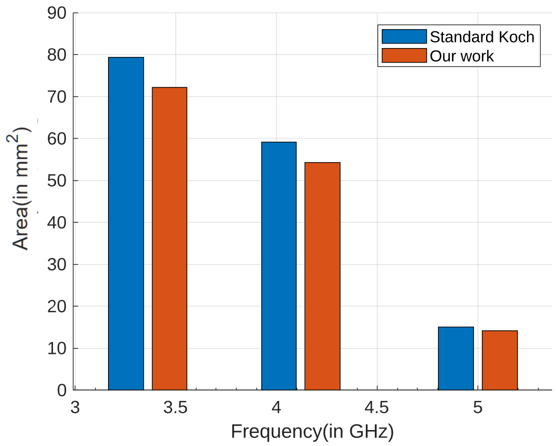

Figure 6: Overall area savings achieved for different antenna designs as a function of center frequency.

Figure 7: Design of three transformed Koch snowflakes for 45°, 60°, and 75°

3. Conclusions

This work introduces a method for expanding the design space of Koch snowflake UWB antennas through an additional design knob (indentation angle) with a procedure for transforming a Koch curve into a polar Koch snowflake. This allows designers to manipulate the indentation angle to alter the frequency characteristics without increasing the antenna footprint. Our measured results have validated the technique and prove that indentation angle is an effective additional design parameter for UWB antennas.

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